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A Comparative Study of Middle School Mathematics Textbooks in China and Russia

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Abstract

This study compares and analyzes the presentation of rational number concepts in two versions of junior high school mathematics textbooks from China and Russia. The findings reveal that while both versions cover core concepts consistently, there are significant differences in their definitional logic and organizational sequence. These differences stem from distinct editorial philosophies: the Russian version emphasizes the intrinsic logic and structural coherence of number system expansion, whereas the Chinese version prioritizes practical knowledge application and cognitive continuity. Based on these findings, the study proposes restructuring the conceptual sequence, clarifying functional positioning, and enhancing the rigor of definitions. It advocates that textbook development should balance logical depth with cognitive accessibility, providing empirical references for cross-cultural mathematics textbook design.

Keywords: Textbook Comparison; Rational Numbers; Extension of Number Fields

1. Introduction

Mathematics textbooks serve as the core medium for knowledge transmission, and their design philosophy profoundly influences students' cognitive structures and logical thinking development. In the era of globalized education, cross-national textbook comparison studies have become an important approach for revealing cultural differences and optimizing curriculum design (Fan et al., 2013). The presentation of foundational mathematical concepts such as number system extensions directly impacts the development of students' algebraic thinking (Van Hoof et al., 2017), while also embodying rich historical content. Consequently, number system extension has become a focal point in comparative textbook research. Internationally, studies on this core concept have long centered on comparisons between Chinese and American, or Chinese and Japanese, textbooks: American materials tend toward “contextualized applications” (such as mathematical

modeling emphasized in PISA assessments), while Japanese textbooks excel in "meticulously designed cognitive progression" (Shimizu, 2009).

However, compared to other cross-national comparisons, comparative research on Chinese and Russian mathematics textbooks remains relatively limited and has yet to form a systematic research framework. Russian mathematics education has profoundly influenced other countries, particularly socialist nations (Alexander and Vogeli, 2010). Since the 1950s, Soviet educational influence on China has been comprehensive and profound. Russian mathematics education is renowned for its axiomatic systems and logical rigor (Karp, 2007), with textbook compilation strictly adhering to the "spiral progression" principle (e.g., Kolmogorov's Introduction to Algebra and Analysis systematically introduces analytical concepts); In contrast, Chinese textbooks emphasize real-world connections and cognitive adaptability (Cai and Jiang, 2017), prioritizing "progressive understanding" through everyday examples (e.g., the Jiangsu Education Press textbook introduces negative numbers via scenarios like temperature and debt). This divergence stems from the deep tension between "formal rigor" and "intuitive perception" in mathematics education (Sfard, 2007), rooted in each nation's educational traditions: Russia inherits Vygotsky's "theory-first" principle (emphasizing top-down transmission of scientific concepts) (Vygotskij, 1979), while China remains profoundly influenced by the Confucian educational philosophy of "gradual progression" (e.g., the cognitive path of "reviewing the old to understand the new" in The Analects) (Li, 2004). The collision of these two approaches offers a unique perspective for mathematics education design.

Existing research indicates fundamental differences in how expanded number systems are presented in textbooks: one approach emphasizes cognitive pathways that reduce students' cognitive load (e.g., making algebraic definitions more intuitive), while the other advocates for structured mathematical logic (Wu, 2011). The representative textbooks selected for this study—China's Su Jiao Edition Mathematics Grade 7 Volume 1 (2024) and Russia's PR Edition Mathematics Grade 6 Volume 2 (2023)—precisely embody these contrasting approaches. Therefore, this paper focuses on the conceptual construction process of rational numbers, conducting a comparative analysis of the relevant conceptual systems. This aims to yield valuable educational insights.

2. Research Subjects

2.1. Selection of Textbooks

The textbooks published by Jiangsu Phoenix Science and Technology Press are widely adopted in one of China's most economically and educationally developed provinces. They embody the principles of China's Compulsory Education Mathematics Curriculum Standards and are renowned for their emphasis on contextual creation, inquiry-based learning, and cognitive adaptability. These textbooks represent the mainstream educational materials following China's "New Curriculum Reform." Textbooks from Prosveshcheniye Publishing House, Russia's largest and most authoritative publisher of basic education materials, are widely used nationwide. This edition inherits the long-standing traditions of Soviet-Russian mathematics education and is

renowned for its logical rigor, systematic theoretical framework, and depth of content. Selecting these two editions signifies the study of "benchmark" products within the mainstream educational systems of both countries. The comparative conclusions thus drawn possess high representativeness and universality, authentically reflecting the typical characteristics of mathematics education in China and Russia.

The seventh-grade upper-semester textbook published by Jiangsu Phoenix Science and Technology Publishing House is hereinafter referred to as the SJ edition textbook, and the sixth-grade lower-semester textbook published by Prosveshcheniye Publishing House is hereinafter referred to as the PR edition textbook.

Characteristics of the educational systems in China and Russia: China has a six-year primary school system, a three-year junior high school system, and a three-year senior high school system; Russia has a four-year primary school system, a five-year junior high school system, and a two-year senior high school system.

Table 1. The specific information selected for textbooks in both countries.

Country	Title	Main Author	Publisher	Publication Date
China	Compulsory education textbook mathematics grade 7 Part 1	Bao Jiansheng	Jiangsu Phoenix Science and Technology Press	2024
Russia	Mathematics Grade 6 Textbook Part 2	N. Ya. Vilenkin, V. I. Zhokhov et al.	Просвещение	2023

2.2. Selection of Research Content

Although the educational systems of the two countries differ slightly (China's seventh-grade first semester corresponds to Russia's sixth-grade second semester), the students are roughly the same age (approximately 13-14 years old) and are at a critical stage where the number system expands from arithmetic to algebra. The "introduction of rational numbers" represents a shared core and challenging component in both curricula. This alignment provides a solid foundation for content synchronization and cognitive development comparability in comparative research. Both countries' textbooks must address the fundamental questions: "How can students grasp the meaning of negative numbers?" and "How can a complete number system (the set of rational numbers) be constructed?" Examining their divergent approaches to this shared problem most effectively reveals underlying differences in instructional philosophy. Since mathematical definitions are crucial for conceptual understanding, this study focuses on the expansion of the number system—a logically interconnected topic in secondary mathematics—specifically analyzing the construction process of the rational number concept through comparative analysis of SJ and PR textbooks. The study will concentrate on comparing the presentation of core concept

definitions, including positive and negative numbers, integers and rational numbers, the number line, opposite numbers, and absolute values.

3. Comparison and Analysis

3.1. Comparison of the Knowledge System and Sequence of Rational Number Concepts

The SJ edition textbook places the content related to rational number concepts in Chapter 2 of the first semester of seventh grade, while the PR edition textbook places this content in Chapter 1 of the second semester of sixth grade (Chapter 4 of the first and second semester textbooks).

Table 2.The specific sequence of rational number concepts in the two editions of the textbook

Sequence	SJ Edition Textbook	PR Edition Textbook
1	2.1 Positive numbers, negative numbers, integers, and rational numbers	4.24 Positive numbers, negative numbers, and the number line
2	2.2 number line	4.25 Opposite numbers and integers
3	2.3.1 absolute value	4.26 mold
4	2.3.2 opposite number	4.35 rational number

Note: The "model" in the PR edition textbook is essentially the absolute value in the People's Education Press edition textbook. In the SJ edition textbook, the concepts of absolute value and opposite number are compiled in Section 2.3. For the sake of comparison and actual teaching arrangements, this article divides them into 2.3.1 and 2.3.2.

A comparative analysis of Table 2 reveals that the two editions of textbooks are highly consistent in terms of the scope of coverage of rational number concepts, but there are significant differences in the organizational structure and presentation order of the knowledge points. This study will focus on two dimensions for in-depth discussion: first, a systematic comparison of the similarities and differences in the definitions of relevant concepts in the two editions of textbooks; second, an in-depth analysis of the underlying causes of differences in the order of knowledge points and their pedagogical significance.

3.2. Comparative Analysis of Definitions of Related Concepts of Rational Numbers

Below, we will conduct a systematic comparative analysis in the order in which the relevant concepts are presented in the SJ edition textbook.

3.2.1. Positive and Negative Numbers

The two editions of the textbook are consistent in their mathematical essence of positive and negative numbers, both using symbols to distinguish the positive and negative properties of numbers, but there are differences in their expressions. The definition of positive and negative numbers in the SJ edition textbook has the following characteristics:

(1) Concise and clear expression, using concrete examples (such as noting that the symbol characteristic of positive numbers is no symbol or "+", and that of negative numbers is "−") to intuitively present the concept;

(2) Diverse example types, covering various numerical forms such as integers and decimals (e.g., +40,000, 1.7, 8,848.86), highlighting the broad applicability of the definition;

(3) International terminology is used, clearly providing the English terms "positive number" and "negative number," enhancing the textbook's international appeal.

In contrast, the PR version of the textbook's definition:

(1) Emphasizes formal characteristics, such as directly defining positive numbers as "numbers with a '+' sign";

(2) The examples are limited to integer forms (e.g., +3, +5) and do not include other types such as decimals;

(3) International terms are not labeled.

Table 3. Concept presentation

SJ Edition Textbook	PR Edition Textbook
Numbers such as 8848.86, 4, +40,000, and 1.7 are positive numbers; numbers such as − 80.97, − 6, −10,000, and−0.6 are negative numbers.	Numbers with the sign "+" are called positive and are written as +3, +5. Numbers with the sign"-"are called negative and are written as −3, −5.

The core commonality between the two versions of the textbook lies in their definitions of positive and negative numbers based on the external symbolic characteristics of numbers. The main differences lie in the richness of examples, the labeling of international terms, and the level of detail in the definitions. Overall, the SJ edition textbook maintains mathematical rigor while emphasizing teaching practicality and an international perspective, which helps students build a more comprehensive understanding of number concepts; the PR edition textbook, on the other hand, emphasizes the formal characteristics of concepts, reflecting a different editorial philosophy.

3.2.2. Integer

There are significant differences between the SJ edition and PR edition textbooks in terms of how they define the concept of integers: The SJ edition uses an extensional definition method, explicitly defining the elements of the integer set through enumeration (positive integers, zero, negative integers). This definition method is intuitive and clear, making it easier for beginners to quickly grasp the concept's extensional meaning, but it relatively weakens the intrinsic connections between mathematical concepts.

The PR edition textbook employs an intension-based definition method, constructing the concept by revealing the logical connection between integers and natural numbers. Specifically, it takes the set of natural numbers as its foundation (in the Russian number system, 0 is not part of

the natural numbers), uses opposite number operations to achieve algebraic expansion, and supplements the zero element to complete the construction of integers. This method emphasizes the generative and systematic nature of mathematical concepts but places higher demands on learners' abstract thinking abilities.

These two distinct writing paradigms reflect two typical orientations in mathematics education: the SJ edition emphasizes the acceptability of knowledge, while the PR edition emphasizes the logical rigor of mathematics.

Table 4. Integer Concept presentation

SJ Edition Textbook	PR Edition Textbook
Among positive numbers, numbers like $+7{,}998$ are called positive integers; among negative numbers, numbers like -9, -998 are called negative integers. Positive integers, negative integers, and zero are collectively referred to as integers. Integers and zero are what we commonly refer to as natural numbers.	Natural numbers, their opposite negative numbers, and zero are called whole numbers.

3.2.3. Rational Number

There are significant differences between the SJ edition textbook and the PR edition textbook in their definitions of rational numbers.

Table 5. Rational Number Concept presentation

SJ Edition Textbook	PR Edition Textbook
Integers and fractions are collectively referred to as rational numbers.	The number that can be expressed as $\frac{p}{q}$,p an integer, and q a natural number, is called a rational number.

The SJ edition defines rational numbers as "the collective term for integers and fractions." This definition is intuitive and helps students grasp the formal characteristics of rational numbers at an early stage. However, since "fractions" lack a clear definition, it is easy for students to become confused about the essence of rational numbers. For example, are $\frac{\pi}{2}$ and $\frac{\sqrt{3}}{2}$ considered "fractions"? If they are considered fractions, it would lead to the erroneous conclusion that they are rational numbers. More seriously, if we let x be any real number, since $x = \frac{x}{1}$, this definition would lead to the fallacy that "any real number is a rational number." This unrestricted definition of "fractions" has caused significant confusion for both teachers and students. The fundamental issue lies in the lack of restrictions on the number domains of the numerator and denominator. In contrast, the definition of rational numbers in the PR edition textbook (which restricts the numerator to integers and the denominator to natural numbers) offers the following advantages:

(1) Rigorous and unambiguous definition: The explicit specification of the range of values for the numerator and denominator allows clear determination based on the definition that $\frac{\pi}{2}$ and $\frac{\sqrt{3}}{2}$ are not rational numbers, effectively eliminating counterexamples.

(2) Revealing the logic of number field expansion:

The definition of rational numbers in the PR edition textbook lays the groundwork for a three-layer construction framework,

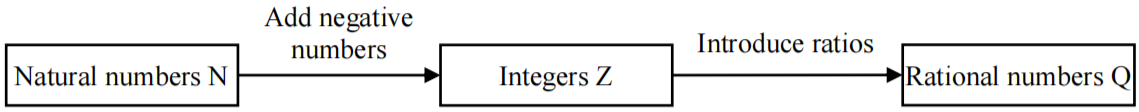

Figure 1. The logic of number field expansion

The system demonstrates the hierarchical construction process of mathematical concepts from simple to complex. This approach focuses on revealing the intrinsic logical chain of number field extensions, guiding students to understand the "construction process of mathematical objects," grasping the essential nature of the generation of mathematical concepts, and facilitating the establishment of core mathematical ideas regarding number field extensions.

(3) Laying the foundation for subsequent learning: This definition provides the conceptual foundation for constructing the rational number field (fractional field) from the integer ring in subsequent algebra and for fractional representations in number theory.

Comparative analysis shows that mathematical textbook definitions must balance rigor (eliminating counterexamples) with logical coherence (reflecting the evolution of concepts). The definition of rational numbers in the PR edition textbook is not only a model of mathematical rigor but also an excellent example of cultivating students' mathematical constructive thinking—it explains rational numbers within the broader framework of field extension. This exposition transcends formalistic descriptions, enabling students to see not isolated concepts but the logical trajectory of the evolution of the entire number system.

3.2.4. Number Axis

The definitions of the number line in the SJ edition and PR edition textbooks are identical in terms of mathematical content: both define it as a straight line with three elements—an origin, a positive direction, and a unit length.

Table 6. Number Axis Concept Presentation

SJ Edition Textbook	PR Edition Textbook
A straight line with a fixed origin, positive direction, and unit length is called a number line (number axis).	The line on which the origin, unit segment, and direction are selected is called the coordinate line. The number that shows the position of a point on the line is called the coordinate of that point.

Note: Unless otherwise specified in the following text, for convenience of expression, the coordinate axes in the PR edition textbooks are sometimes referred to as number lines.

The main differences between the two editions are as follows:

(1) Different sequencing: The SJ edition textbook places the number line after the study of rational numbers, teaching it as an independent section; The PR edition textbook integrates the number line with positive and negative numbers into the same section.

(2) Different functional focuses: The SJ edition textbook emphasizes "representing numbers with points," using points on the line to represent rational numbers. Its core objective is to establish a geometric representation of numbers, leveraging the number line's intuitiveness to aid understanding of rational number concepts, thereby embodying the principle of combining numbers and shapes. The PR edition textbook emphasizes "using numbers to represent points," stressing the use of numbers to indicate the position of points. Its focus is on constructing a coordinate system for points, placing greater emphasis on the geometric applications of rational numbers.

Overall, introducing the number line as a geometric representation and application tool for numbers aligns with students' cognitive patterns and facilitates an intuitive understanding of positive and negative numbers. Both editions utilize the number line to highlight the two dimensions of positive and negative numbers: the "essence" (understanding of the conceptual core) and the "application" (practical application skills). The distinctive features of both versions in terms of arrangement and functional positioning are worthy of mutual reference and dialectical unity, aiming to help students grasp the concepts of positive and negative numbers more comprehensively and deeply, enhance their application abilities, and thereby optimize the logical coherence and practical effectiveness of the textbook system.

3.2.5. Opposite Number

The two editions of the textbook are highly consistent in their definitions of the concept of opposite numbers. The main difference lies in the order of arrangement of the number line content: the SJ edition places it after the study of rational numbers, while the PR edition places it before the introduction of rational numbers. This difference in order reflects the underlying differences in their functional orientations.

Table 7. Opposite Number Concept Presentation

SJ Edition Textbook	PR Edition Textbook
Two numbers that differ only in sign are called opposites.	Two numbers that differ only in sign are called opposites.

SJ Edition Textbook (Rational Numbers → Opposite Numbers): This approach aims to use the concepts of opposite numbers and absolute value to highlight the core structure of rational numbers—the "sign" and "length" (absolute value)—enabling students to thoroughly understand

the composition of rational numbers. This path follows a logical progression from the general (integers) to the specific (the property of opposite numbers), emphasizing structural analysis of rational numbers.

PR Edition Textbook (opposite numbers → integers): In addition to the structural analysis function mentioned above, its core role lies in using the concept of opposite numbers to construct (or define) the set of integers. This highlights the foundational role of opposite numbers in the expansion of the number system, following a logical progression from the specific (opposite number relationships) to the general (constructing the set of integers), with a focus on the generation and expansion of the number system.

3.2.6. Absolute Value

Both the SJ edition and PR edition textbooks define the core concept of absolute value using a geometric interpretation: the distance between the point representing the number on the number line and the origin. This definition is intuitive and visual, making it easier for students to establish a foundational understanding when first encountering the concept.

Table 8. Absolute Value Concept presentation

SJ Edition Textbook	PR Edition Textbook
Generally speaking, the distance between a point representing a number on a number line and the origin is called the absolute value of that number. The absolute value of a number is denoted as |a| and read as "the absolute value of a."	The module of the number n is defined as the distance (in unit segments) from the origin to point N(n). It is written as $\|n\|$. The definition of the modulus can also be written as follows: $\|n\| = n, if\ n > 0,$ $\|n\| = 0, if\ n = 0,$ $\|n\| = -n, if\ n < 0,$

However, the PR edition textbook is more rigorous in its conceptual formulation. It explicitly specifies that this "distance" is the result of measuring "in units of length," rather than an actual physical measurement. This clarification effectively avoids ambiguity—for example, if the same distance is measured using centimeters and decimeters as units, the numerical representations will inevitably differ. This rigor reflects the PR edition textbook's approach to rational number instruction: guiding students to view rational numbers as independent, abstract algebraic objects rather than geometric entities. This approach is evident in its definition method: it not only provides a geometric explanation but also supplements it with the algebraic definition of absolute value.

In contrast, the SJ edition textbook adopts only the geometric meaning as the sole formal definition. The potential issue with this singular geometric definition approach is that it may blur the essential distinction between "numbers" (algebraic objects) and "distance" (metric quantities).

Although the two are highly compatible in the number line model, they belong to different conceptual categories. The essence of the integration of numbers and shapes lies in establishing connections rather than confusing categories. Anchoring the formal definition of algebraic concepts (numbers) entirely to geometric concepts (distance) may facilitate intuitive entry, but it may hinder students' deep understanding of absolute value as an independent, abstract algebraic object.

4. Conclusions and Implications

4.1. Conclusions

Through a comparative analysis of the rational number-related concepts in the SJ and PR editions of textbooks, it was found that the two editions are highly consistent in their coverage of core concepts, with the main differences lying in the way concepts are defined and the logical sequence in which they are presented. These differences fundamentally reflect two distinct textbook development philosophies: the PR edition emphasizes the logical consistency of mathematical knowledge and the intrinsic connections between concepts, focusing on the constructive logic of number system expansion; whereas the SJ edition prioritizes the intuitive presentation of knowledge and its immediate applicability, aiming to reduce cognitive load for beginners.

The comparative analysis indicates that the PR edition textbook has significant advantages in terms of the rigor of concept expression and the structural integrity of the knowledge system, providing important reference value for the compilation and instructional design of the SJ edition textbook. Meanwhile, the successful experiences of the SJ edition in terms of the intuitive nature of concept introduction and cognitive adaptability also offer valuable insights for the improvement of the PR edition textbook.

4.2. Implications

The definition of rational number-related concepts should not only focus on their representational forms but also reveal their underlying logical relationships. Based on the comparative analysis of the two versions of textbooks and their respective advantages, the following three suggestions are proposed for the presentation of rational numbers and their sequence concepts:

4.2.1. Optimizing the Presentation Sequence of Related Concepts

Suggested sequence: positive numbers, negative numbers, 0 → number line → opposite numbers → integers → rational numbers → absolute value. Placing the number line after positive and negative numbers and before rational numbers facilitates students' geometric understanding of positive and negative numbers. Introducing opposite numbers before integers helps reveal the constructive logic of expanding the number system.

4.2.2. Clarify the functional positioning of concepts

When introducing concepts, emphasize their core functions:

(1) Number line: serves as the geometric representation of rational numbers, achieving the integration of numbers and shapes, and aiding in the understanding of positive and negative numbers.

(2) Opposite numbers: highlight the symbolic characteristics of rational numbers, serving as a key link in the expansion of the natural number system to the integer system.

(3) Integers: form the basic elements of the rational number system.

4.2.3. Strengthening the Rigorous Definition of Concepts

(1) Definitions of positive and negative numbers: It is recommended to use property-based definitions (e.g., positive numbers are numbers greater than 0).

(2) Integers: The collective term for natural numbers and their opposite numbers.

(3) Rational numbers: Numbers that can be expressed in the form of a fraction p/q (where p is an integer and q is a non-zero integer).

(4) Absolute value: It is recommended to use an algebraic definition as the main approach, with a geometric interpretation as a supplement.

Through a comparative analysis of the rational number concepts in the SJ and PR editions of textbooks, this study clearly highlights the differences in the two textbooks' editorial philosophies and their respective values: the PR edition excels in logical rigor and system construction, while the SJ edition excels in intuitive introduction and cognitive adaptation. This difference is not a matter of superiority or inferiority but provides complementary perspectives for textbook optimization. Based on this, the study further proposes specific recommendations for optimizing the sequence of concept presentation, clarifying core functional positioning, and strengthening the rigor of definitions. The core of these recommendations lies in guiding textbook compilation and teaching practices to place greater emphasis on revealing the intrinsic logical connections of concepts while also considering students' cognitive starting points. Future rational number concept instruction should actively draw on the essence of both editions of the textbook, seeking a better balance between logical depth and cognitive breadth to promote students' formation of a profound and accurate understanding of the rational number system.

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